Attorney Docket No.: 018158-021800US
Client Reference No.: VX-1166

PATENT APPLICATION

WAVEFRONT RECONSTRUCTION USING A FOURIER TRANSFORMATION ALGORITHM

Inventors: DIMITRI CHERNYAK,
a citizen of The United States of America, residing at
781 North. Fair Oaks Avenue #6
Sunnyvale, California 94085

CHARLES E. CAMPBELL,
a citizen of The United States of America, residing at
2908 Elmwood Court
Berkeley, California 94705

ERIK GROSS,
a citizen of The United States of America, residing at
923 Ilma Way
Palo Alto, California 94306

SEEMA SOMANI,
a citizen of India, residing at
903 Erie Circle
Milpitas, California 95035

Assignee: VISX, Inc.
3400 Central Expressway
Santa Clara, CA, 95051-0703

Entity: Large

TOWNSEND and TOWNSEND and CREW LLP
Two Embarcadero Center, 8th Floor
San Francisco, California 94111-3834
Tel: 650-326-2400

WAVEFRONT RECONSTRUCTION USING A FOURIER TRANSFORMATION ALGORITHM

BACKGROUND OF THE INVENTION

[0001] The present invention generally relates to measuring optical errors of optical systems. More particularly, the invention relates to improved methods and systems for reconstructing a wavefront surface/elevation map of optical tissues of an eye and to improved systems for calculating an ablation pattern.

[0002] Known laser eye surgery procedures generally employ an ultraviolet or infrared laser to remove a microscopic layer of stromal tissue from the cornea of the eye. The laser typically removes a selected shape of the corneal tissue, often to correct refractive errors of the eye. Ultraviolet laser ablation results in photodecomposition of the corneal tissue, but generally does not cause significant thermal damage to adjacent and underlying tissues of the eye. The irradiated molecules are broken into smaller volatile fragments photochemically, directly breaking the intermolecular bonds.

[0003] Laser ablation procedures can remove the targeted stroma of the cornea to change the cornea's contour for varying purposes, such as for correcting myopia, hyperopia, astigmatism, and the like. Control over the distribution of ablation energy across the cornea may be provided by a variety of systems and methods, including the use of ablatable masks, fixed and moveable apertures, controlled scanning systems, eye movement tracking mechanisms, and the like. In known systems, the laser beam often comprises a series of discrete pulses of laser light energy, with the total shape and amount of tissue removed being determined by the shape, size, location, and/or number of laser energy pulses impinging on the cornea. A variety of algorithms may be used to calculate the pattern of laser pulses used to reshape the cornea so as to correct a refractive error of the eye. Known systems make use of a variety of forms of lasers and/or laser energy to effect the correction, including infrared lasers, ultraviolet lasers, femtosecond lasers, wavelength multiplied solid-state lasers, and the like. Alternative vision correction techniques make use of radial incisions in the cornea, intraocular lenses, removable corneal support structures, and the like.

[0004] Known corneal correction treatment methods have generally been successful in correcting standard vision errors, such as myopia, hyperopia, astigmatism, and the like. However, as with all successes, still further improvements would be desirable.

Toward that end, wavefront measurement systems are now available to accurately measure the refractive characteristics of a particular patient's eye. One exemplary wavefront technology system is the VISX WaveScan® System, which uses a Hartmann-Shack wavefront lenslet array that can quantify aberrations throughout the entire optical system of the patient's eye, including first- and second-order sphero-cylindrical errors, coma, and third and fourth-order aberrations related to coma, astigmatism, and spherical aberrations.

[0005] Wavefront measurement of the eye may be used to create a high order aberration map or wavefront elevation map that permits assessment of aberrations throughout the optical pathway of the eye, e.g., both internal aberrations and aberrations on the corneal surface. The aberration map may then be used to compute a custom ablation pattern for allowing a surgical laser system to correct the complex aberrations in and on the patient's eye. Known methods for calculation of a customized ablation pattern using wavefront sensor data generally involves mathematically modeling an optical surface of the eye using expansion series techniques. More specifically, Zernike polynomials have been employed to model the optical surface, as proposed by Liang et al., in *Objective Measurement of Wave Aberrations of the Human Eye with the Use of a Harman-Shack Wave-front Sensor*, Journal Optical Society of America, Jul. 1994, vol. 11, No. 7, pp. 1-9, the entire contents of which is hereby incorporated by reference. Coefficients of the Zernike polynomials are derived through known fitting techniques, and the refractive correction procedure is then determined using the shape of the optical surface of the eye, as indicated by the mathematical series expansion model.

[0006] The Zernike function method of surface reconstruction and its accuracy for normal eyes have been studied extensively for regular corneal shapes. *See* Schweigerling, J., and Grievenkamp, J.E., "*Using corneal height maps and polynomial decomposition to determine corneal aberrations,*" Opt. Vis. Sci., Vol. 74, No. 11 (1997) and Gurao, A. and Artal, P., "*Corneal wave aberration from videokeratography: Accuracy and limitations of the procedure*," JOSAA, Vol. 17, No. 6 (2000). Some commentators have shown that the 6th order Zernike polynomial reconstruction method provide an inferior fit when compared to a method of Bhatia-Wolf polynomials. *See* D.R. Ishkander et al., "*An Alternative Polynomial Representation of the Wavefront Error Function*," IEEE Transations on Biomedical Engineering, Vol. 49, No. 4, (2002). The discrepancy was most significant for eyes with a keratoconus condition. It has further been suggested that the known Zernike polynomial modeling methods may result in errors or "noise" which can lead to a less than ideal refractive correction. Furthermore, the known surface modeling techniques are

somewhat indirect, and may lead to unnecessary errors in calculation, as well as a lack of understanding of the physical correction to be performed.

[0007] Therefore, in light of above, it would be desirable to provide improved methods and systems for mathematically modeling optical tissues of an eye.

BRIEF SUMMARY OF THE INVENTION

[0008] The present invention provides systems, software, and methods for measuring errors and reconstructing wavefront elevation maps in an optical system using Fourier transform algorithms.

[0009] In one aspect, the present invention provides a method of reconstructing optical tissues of an eye. The method comprises transmitting an image through the optical tissues of the eye. Surface gradients from the transmitted image are measured across the optical tissues of the eye. A Fourier transform algorithm is applied to the surface gradients to reconstruct a surface that corresponds to the optical tissues of the eye.

[0010] In some embodiments, the measurement of the surface gradient data is carried out with a Hartmann-Shack sensor assembly. The image is transmitted by the optical tissues as a plurality of beamlets and the surface gradients will be in the form of an array of gradients. Each gradient corresponds to an associated portion of the optical tissues of the eye and each beamlet is transmitted through the optical tissues according to the corresponding gradient. In such embodiments, the measured surface will be in the form of a wavefront surface or wavefront elevation map.

[0011] In one embodiment, the Fourier transformation algorithm may apply a fast Fourier transform or a discrete Fourier decomposition and an inverse discrete Fourier transform. Some Fourier transform algorithms may include a mean gradient field so as to remove a tilt from the reconstructed surface. Unlike Zernike polynomial reconstruction, which is based on a unit circle, the Fourier transformation uses all of the available information in the reconstruction.

[0012] A computed correction ablation pattern based on the reconstructed optical tissues of the eye, as indicated by the Fourier reconstructed surface, may be calculated and aligned with the eye. The correction ablation pattern typically comprises a proposed change in elevations of the optical tissue so as to effect a desired change in optical properties of the eye. After the correction ablation pattern is derived, laser ablation may be used to modify the optical tissue surface.

[0013] In another aspect, the present invention provides a method for measuring optical tissues of an eye. The method comprises transmitting an image through the optical tissues. Local gradients across the optical tissues are determined from the transmitted image. A wavefront error of the eye is mapped by applying a Fourier transform algorithm to the surface gradients across the optical tissues of the eye.

[0014] Measurement of the local gradients may be carried out by a Hartmann-Shack sensor assembly. A mean gradient field may be added to the wavefront error to correct for tilt. After the wavefront error is mapped, a laser ablation treatment table may be created based on the mapped wavefront error of the optical tissues of the eye and the optical tissue surface may be modified according to the correction ablation pattern by laser ablation.

[0015] In another aspect, the present invention further provides a system for measuring a wavefront error of optical tissue. The system comprises a memory coupled to a processor. The memory may be configured to store a plurality of code modules for execution by the processor. The plurality of code modules comprise a module for transmitting an image through the optical tissues, a module for determining local gradients across the optical tissues from the transmitted image, and a module for mapping a wavefront error of the eye by applying a Fourier transform algorithm to the surface gradients across the optical tissues of the eye.

[0016] The system may include an image source coupled to the processor for transmitting a source image through the optical tissues of the eye. The image may be a point or small spot of light, or any other suitable image. The system may also include a wavefront sensor system coupled to the processor, such as a Hartmann-Shack sensor assembly.

[0017] The system may include one or more cameras to track the position of the optical tissues. In such embodiments, the plurality of code modules also includes a module for registering the wavefront error relative to an image of the optical tissues that was obtained by the camera(s).

[0018] In some embodiments, the system may include a module for calculating a customized laser ablation program or ablation table based on the reconstructed surface of the optical tissues. A laser system may be in communication with the above measurement system. The laser system may include a laser that is programmable to deliver a laser energy to the optical tissues according to the correction ablation pattern.

[0019] In a further aspect, the present invention provides a computer program stored on a computer-readable storage medium. The computer program comprises code for transmitting an image through the optical tissues of the eye, code for measuring surface

gradients from the transmitted image across the optical tissues of the eye, and code for mapping a wavefront error of the eye by applying a Fourier transform algorithm to the surface gradients across the optical tissues of the eye.

[0020] In some embodiments, the computer program may include code for computing an ablation pattern based on the optical tissues of the eye as indicated by the Fourier reconstructed surface, code for controlling the delivery of laser energy to the optical tissues according to the correction ablation pattern, and/or code for aligning the mapped wavefront error with an image of the optical tissues of the eye.

[0021] These and other aspects will be apparent in the remainder of the figures, description and claims.

BRIEF DESCRIPTION OF THE DRAWINGS

[0022] Figure 1 illustrates a laser ablation system according to an embodiment of the present invention.

[0023] Figure 2 illustrates a simplified computer system according to an embodiment of the present invention.

[0024] Figure 3 illustrates a wavefront measurement system according to an embodiment of the present invention.

[0025] Figure 3A illustrates another wavefront measurement system according to another embodiment of the present invention.

[0026] Figure 4 schematically illustrates a simplified set of modules that carry out one method of the present invention.

[0027] Figure 5 is a flow chart that schematically illustrates a method of using a Fourier transform algorithm to determine a corneal ablation treatment program.

[0028] Figure 6 schematically illustrates a comparison of a direct integration reconstruction, a 6th order Zernike polynomial reconstruction, a 10th order Zernike polynomial reconstruction, and a Fourier transform reconstruction for a surface having a +2 ablation on a 6 mm pupil.

[0029] Figure 7 schematically illustrates a comparison of a direct integration reconstruction, a 6th order Zernike polynomial reconstruction, a 10th order Zernike polynomial reconstruction, and a Fourier transform reconstruction for a presbyopia surface.

[0030] Figure 8 schematically illustrates a comparison of a direct integration reconstruction, a 6th order Zernike polynomial reconstruction, a 10th order Zernike

polynomial reconstruction, and a Fourier transform reconstruction for another presbyopia surface.

[0031] Figure 9 illustrates a difference in a gradient field calculated from a reconstructed wavefront from a Fourier transform reconstruction algorithm (F Gradient), Zernike polynomial reconstruction algorithm (Z Gradient), a direct integration reconstruction algorithm (D Gradient) and a directly measured wavefront.

[0032] Figure 10 illustrates intensity plots of reconstructed wavefronts for Fourier, 10th Order Zernike and Direct Integration reconstructions.

[0033] Figure 11 illustrates intensity plots of reconstructed wavefronts for Fourier, 6th Order Zernike and Direct Integration reconstructions.

DETAILED DESCRIPTION OF THE INVENTION

[0034] The present invention provides systems, software, and methods that use a high speed and accurate Fourier transformation algorithm to mathematically reconstruct optical tissues of an eye.

[0035] The present invention is generally useful for enhancing the accuracy and efficacy of laser eye surgical procedures, such as photorefractive keratectomy (PRK), phototherapeutic keratectomy (PTK), laser in situ keratomileusis (LASIK), and the like. The present invention can provide enhanced optical accuracy of refractive procedures by improving the methodology for measuring the optical errors of the eye and hence calculate a more accurate refractive ablation program. In one particular embodiment, the present invention is related to therapeutic wavefront-based ablations of pathological eyes.

[0036] The present invention can be readily adapted for use with existing laser systems, wavefront measurement systems, and other optical measurement devices. By providing a more direct (and hence, less prone to noise and other error) methodology for measuring and correcting errors of an optical system, the present invention may facilitate sculpting of the cornea so that treated eyes regularly exceed the normal 20/20 threshold of desired vision.

[0037] While the systems, software, and methods of the present invention are described primarily in the context of a laser eye surgery system, it should be understood the present invention may be adapted for use in alternative eye treatment procedures and systems such as spectacle lenses, intraocular lenses, contact lenses, corneal ring implants, collagenous corneal tissue thermal remodeling, and the like.

[0038] Referring now to Figure 1, a laser eye surgery system 10 of the present invention includes a laser 12 that produces a laser beam 14. Laser 12 is optically coupled to laser delivery optics 16, which directs laser beam 14 to an eye of patient P. A delivery optics support structure (not shown here for clarity) extends from a frame 18 supporting laser 12. A microscope 20 is mounted on the delivery optics support structure, the microscope often being used to image a cornea of the eye.

[0039] Laser 12 generally comprises an excimer laser, ideally comprising an argon-fluorine laser producing pulses of laser light having a wavelength of approximately 193 nm. Laser 12 will preferably be designed to provide a feedback stabilized fluence at the patient's eye, delivered via laser delivery optics 16. The present invention may also be useful with alternative sources of ultraviolet or infrared radiation, particularly those adapted to controllably ablate the corneal tissue without causing significant damage to adjacent and/or underlying tissues of the eye. In alternate embodiments, the laser beam source employs a solid state laser source having a wavelength between 193 and 215 nm as described in U.S. Patents Nos. 5,520,679, and 5,144,630 to Lin and 5,742,626 to Mead, the full disclosures of which are incorporated herein by reference. In another embodiment, the laser source is an infrared laser as described in U.S. Patent Nos. 5,782,822 and 6,090,102 to Telfair, the full disclosures of which are incorporated herein by reference. Hence, although an excimer laser is the illustrative source of an ablating beam, other lasers may be used in the present invention.

[0040] Laser 12 and laser delivery optics 16 will generally direct laser beam 14 to the eye of patient P under the direction of a computer system 22. Computer system 22 will often selectively adjust laser beam 14 to expose portions of the cornea to the pulses of laser energy so as to effect a predetermined sculpting of the cornea and alter the refractive characteristics of the eye. In many embodiments, both laser 12 and the laser delivery optical system 16 will be under control of computer system 22 to effect the desired laser sculpting process, with the computer system effecting (and optionally modifying) the pattern of laser pulses. The pattern of pulses may be summarized in machine readable data of tangible media 29 in the form of a treatment table, and the treatment table may be adjusted according to feedback input into computer system 22 from an automated image analysis system (or manually input into the processor by a system operator) in response to real-time feedback data provided from an ablation monitoring system feedback system. The laser treatment system 10, and computer system 22 may continue and/or terminate a sculpting treatment in

response to the feedback, and may optionally also modify the planned sculpting based at least in part on the feedback.

[0041] Additional components and subsystems may be included with laser system 10, as should be understood by those of skill in the art. For example, spatial and/or temporal integrators may be included to control the distribution of energy within the laser beam, as described in U.S. Patent No. 5,646,791, the disclosure of which is incorporated herein by reference. An ablation effluent evacuator/filter, and other ancillary components of the laser surgery system which are not necessary to an understanding of the invention, need not be described in detail for an understanding of the present invention.

[0042] Figure 2 is a simplified block diagram of an exemplary computer system 22 that may be used by the laser surgical system 10 of the present invention. Computer system 22 typically includes at least one processor 52 which may communicate with a number of peripheral devices via a bus subsystem 54. These peripheral devices may include a storage subsystem 56, comprising a memory subsystem 58 and a file storage subsystem 60, user interface input devices 62, user interface output devices 64, and a network interface subsystem 66. Network interface subsystem 66 provides an interface to outside networks 68 and/or other devices, such as the wavefront measurement system 30.

[0043] User interface input devices 62 may include a keyboard, pointing devices such as a mouse, trackball, touch pad, or graphics tablet, a scanner, foot pedals, a joystick, a touchscreen incorporated into the display, audio input devices such as voice recognition systems, microphones, and other types of input devices. User input devices 62 will often be used to download a computer executable code from a tangible storage media 29 embodying any of the methods of the present invention. In general, use of the term “input device” is intended to include a variety of conventional and proprietary devices and ways to input information into computer system 22.

[0044] User interface output devices 64 may include a display subsystem, a printer, a fax machine, or non-visual displays such as audio output devices. The display subsystem may be a cathode ray tube (CRT), a flat-panel device such as a liquid crystal display (LCD), a projection device, or the like. The display subsystem may also provide a non-visual display such as via audio output devices. In general, use of the term “output device” is intended to include a variety of conventional and proprietary devices and ways to output information from computer system 22 to a user.

[0045] Storage subsystem 56 stores the basic programming and data constructs that provide the functionality of the various embodiments of the present invention.

For example, a database and modules implementing the functionality of the methods of the present invention, as described herein, may be stored in storage subsystem 56. These software modules are generally executed by processor 52. In a distributed environment, the software modules may be stored on a plurality of computer systems and executed by processors of the plurality of computer systems. Storage subsystem 56 typically comprises memory subsystem 58 and file storage subsystem 60.

[0046] Memory subsystem 58 typically includes a number of memories including a main random access memory (RAM) 70 for storage of instructions and data during program execution and a read only memory (ROM) 72 in which fixed instructions are stored. File storage subsystem 60 provides persistent (non-volatile) storage for program and data files, and may include tangible storage media 29 (Figure 1) which may optionally embody wavefront sensor data, wavefront gradients, a wavefront elevation map, a treatment map, and/or an ablation table. File storage subsystem 60 may include a hard disk drive, a floppy disk drive along with associated removable media, a Compact Digital Read Only Memory (CD-ROM) drive, an optical drive, DVD, CD-R, CD-RW, solid-state removable memory, and/or other removable media cartridges or disks. One or more of the drives may be located at remote locations on other connected computers at other sites coupled to computer system 22. The modules implementing the functionality of the present invention may be stored by file storage subsystem 60.

[0047] Bus subsystem 54 provides a mechanism for letting the various components and subsystems of computer system 22 communicate with each other as intended. The various subsystems and components of computer system 22 need not be at the same physical location but may be distributed at various locations within a distributed network. Although bus subsystem 54 is shown schematically as a single bus, alternate embodiments of the bus subsystem may utilize multiple busses.

[0048] Computer system 22 itself can be of varying types including a personal computer, a portable computer, a workstation, a computer terminal, a network computer, a control system in a wavefront measurement system or laser surgical system, a mainframe, or any other data processing system. Due to the ever-changing nature of computers and networks, the description of computer system 22 depicted in Figure 2 is intended only as a specific example for purposes of illustrating one embodiment of the present invention. Many other configurations of computer system 22 are possible having more or less components than the computer system depicted in Figure 2.

[0049] Referring now to Figure 3, one embodiment of a wavefront measurement system 30 is schematically illustrated in simplified form. In very general terms, wavefront measurement system 30 is configured to sense local slopes of a gradient map exiting the patient's eye. Devices based on the Hartmann-Shack principle generally include a lenslet array to sample the gradient map uniformly over an aperture, which is typically the exit pupil of the eye. Thereafter, the local slopes of the gradient map are analyzed so as to reconstruct the wavefront surface or map.

[0050] More specifically, one wavefront measurement system 30 includes an image source 32, such as a laser, which projects a source image through optical tissues 34 of eye E so as to form an image 44 upon a surface of retina R. The image from retina R is transmitted by the optical system of the eye (e.g., optical tissues 34) and imaged onto a wavefront sensor 36 by system optics 37. The wavefront sensor 36 communicates signals to a computer system 22' for measurement of the optical errors in the optical tissues 34 and/or determination of an optical tissue ablation treatment program. Computer 22' may include the same or similar hardware as the computer system 22 illustrated in Figures 1 and 2. Computer system 22' may be in communication with computer system 22 that directs the laser surgery system 10, or some or all of the components of computer system 22, 22' of the wavefront measurement system 30 and laser surgery system 10 may be combined or separate. If desired, data from wavefront sensor 36 may be transmitted to a laser computer system 22 via tangible media 29, via an I/O port, via an networking connection 66 such as an intranet or the Internet, or the like.

[0051] Wavefront sensor 36 generally comprises a lenslet array 38 and an image sensor 40. As the image from retina R is transmitted through optical tissues 34 and imaged onto a surface of image sensor 40 and an image of the eye pupil P is similarly imaged onto a surface of lenslet array 38, the lenslet array separates the transmitted image into an array of beamlets 42, and (in combination with other optical components of the system) images the separated beamlets on the surface of sensor 40. Sensor 40 typically comprises a charged couple device or "CCD," and senses the characteristics of these individual beamlets, which can be used to determine the characteristics of an associated region of optical tissues 34. In particular, where image 44 comprises a point or small spot of light, a location of the transmitted spot as imaged by a beamlet can directly indicate a local gradient of the associated region of optical tissue.

[0052] Eye E generally defines an anterior orientation ANT and a posterior orientation POS. Image source 32 generally projects an image in a posterior orientation

through optical tissues 34 onto retina R as indicated in Figure 3. Optical tissues 34 again transmit image 44 from the retina anteriorly toward wavefront sensor 36. Image 44 actually formed on retina R may be distorted by any imperfections in the eye's optical system when the image source is originally transmitted by optical tissues 34. Optionally, image source projection optics 46 may be configured or adapted to decrease any distortion of image 44.

[0053] In some embodiments, image source optics 46 may decrease lower order optical errors by compensating for spherical and/or cylindrical errors of optical tissues 34. Higher order optical errors of the optical tissues may also be compensated through the use of an adaptive optic element, such as a deformable mirror (described below). Use of an image source 32 selected to define a point or small spot at image 44 upon retina R may facilitate the analysis of the data provided by wavefront sensor 36. Distortion of image 44 may be limited by transmitting a source image through a central region 48 of optical tissues 34 which is smaller than a pupil 50, as the central portion of the pupil may be less prone to optical errors than the peripheral portion. Regardless of the particular image source structure, it will be generally be beneficial to have a well-defined and accurately formed image 44 on retina R.

[0054] In one embodiment, the wavefront data may be stored in a computer readable medium 29 or a memory of the wavefront sensor system 30 in two separate arrays containing the x and y wavefront gradient values obtained from image spot analysis of the Hartmann-Shack sensor images, plus the x and y pupil center offsets from the nominal center of the Hartmann-Shack lenslet array, as measured by the pupil camera 51 (Figure 3) image. Such information contains all the available information on the wavefront error of the eye and is sufficient to reconstruct the wavefront or any portion of it. In such embodiments, there is no need to reprocess the Hartmann-Shack image more than once, and the data space required to store the gradient array is not large. For example, to accommodate an image of a pupil with an 8 mm diameter, an array of a 20 x 20 size (i.e., 400 elements) is often sufficient. As can be appreciated, in other embodiments, the wavefront data may be stored in a memory of the wavefront sensor system in a single array or multiple arrays.

[0055] While the methods of the present invention will generally be described with reference to sensing of an image 44, it should be understood that a series of wavefront sensor data readings may be taken. For example, a time series of wavefront data readings may help to provide a more accurate overall determination of the ocular tissue aberrations. As the ocular tissues can vary in shape over a brief period of time, a plurality of temporally separated wavefront sensor measurements can avoid relying on a single snapshot of the

optical characteristics as the basis for a refractive correcting procedure. Still further alternatives are also available, including taking wavefront sensor data of the eye with the eye in differing configurations, positions, and/or orientations. For example, a patient will often help maintain alignment of the eye with wavefront measurement system 30 by focusing on a fixation target, as described in U.S. Patent No. 6,004,313, the full disclosure of which is incorporated herein by reference. By varying a position of the fixation target as described in that reference, optical characteristics of the eye may be determined while the eye accommodates or adapts to image a field of view at a varying distance and/or angles.

[0056] The location of the optical axis of the eye may be verified by reference to the data provided from a pupil camera 52. In the exemplary embodiment, a pupil camera 52 images pupil 50 so as to determine a position of the pupil for registration of the wavefront sensor data relative to the optical tissues.

[0057] An alternative embodiment of a wavefront measurement system is illustrated in Figure 3A. The major components of the system of Figure 3A are similar to those of Figure 3. Additionally, Figure 3A includes an adaptive optical element 53 in the form of a deformable mirror. The source image is reflected from deformable mirror 98 during transmission to retina R, and the deformable mirror is also along the optical path used to form the transmitted image between retina R and imaging sensor 40. Deformable mirror 98 can be controllably deformed by computer system 22 to limit distortion of the image formed on the retina or of subsequent images formed of the images formed on the retina, and may enhance the accuracy of the resultant wavefront data. The structure and use of the system of Figure 3A are more fully described in U.S. Patent No. 6,095,651, the full disclosure of which is incorporated herein by reference.

[0058] The components of an embodiment of a wavefront measurement system for measuring the eye and ablations comprise elements of a VISX WaveScan®, available from VISX, INCORPORATED of Santa Clara, California. One embodiment includes a WaveScan® with a deformable mirror as described above. An alternate embodiment of a wavefront measuring system is described in U. S. Patent No. 6,271,915, the full disclosure of which is incorporated herein by reference.

[0059] Figure 4 schematically illustrates a simplified set of modules for carrying out a method according to one embodiment of the present invention. The modules may be software modules on a computer readable medium that is processed by processor 52 (Figure 2), hardware modules, or a combination thereof. A wavefront aberration module 80 typically receives data from the wavefront sensors and measures the aberrations and other

optical characteristics of the entire optical tissue system imaged. The data from the wavefront sensors are typically generated by transmitting an image (such as a small spot or point of light) through the optical tissues, as described above. Wavefront aberration module 80 produces an array of optical gradients or a gradient map. The optical gradient data from wavefront aberration module 80 may be transmitted to a Fourier transform module 82, where an optical surface or a wavefront elevation surface map is mathematically reconstructed from the optical gradient data.

[0060] It should be understood that the optical surface need not precisely match an actual tissue surface, as the gradient data will show the effects of aberrations which are actually located throughout the ocular tissue system. Nonetheless, corrections imposed on an optical tissue surface so as to correct the aberrations derived from the gradients should correct the optical tissue system. As used herein terms such as "an optical tissue surface" may encompass a theoretical tissue surface (derived, for example, from wavefront sensor data), an actual tissue surface, and/or a tissue surface formed for purposes of treatment (for example, by incising corneal tissues so as to allow a flap of the corneal epithelium and stroma to be displaced and expose the underlying stroma during a LASIK procedure).

[0061] Once the wavefront elevation surface map is generated by Fourier transform module 82, the wavefront gradient map may be transmitted to a laser treatment module 84 for generation of a laser ablation treatment to correct for the optical errors in the optical tissues.

[0062] Figure 5 is a detailed flow chart which illustrates a data flow and method steps of one Fourier based method of generating a laser ablation treatment. The illustrated method is typically carried out by a system that includes a processor and a memory coupled to the processor. The memory may be configured to store a plurality of modules which have the instructions and algorithms for carrying out the steps of the method.

[0063] As can be appreciated, the present invention should not be limited to the order of steps, or the specific steps illustrated, and various modifications to the method, such as having more or less steps, may be made without departing from the scope of the present invention. For comparison purposes, a series expansion method of generating a wavefront elevation map is shown in dotted lines, and are optional steps.

[0064] A wavefront measurement system that includes a wavefront sensor (such as a Hartmann-Shack sensor) may be used to obtain one or more displacement maps 90 (e.g., Hartmann-Shack displacement maps) of the optical tissues of the eye. The

displacement map may be obtained by transmitting an image through the optical tissues of the eye and sensing the exiting wavefront surface.

[0065] From the displacement map 90, it is possible to calculate a surface gradient or gradient map 92 (e.g., Hartmann-Shack gradient map) across the optical tissues of the eye. Gradient map 92 may comprise an array of the localized gradients as calculated from each location for each lenslet of the Hartmann-Shack sensor.

[0066] A Fourier transform may be applied to the gradient map to mathematically reconstruct the optical tissues. The Fourier transform will typically output the reconstructed optical tissue in the form of a wavefront elevation map.

[0067] It has been found that a Fourier transform reconstruction method, such as a fast Fourier transformation (FFT), is many times faster than currently used 6th order Zernike or polynomial reconstruction methods and yields a more accurate reconstruction of the actual wavefront. Advantageously, the Fourier reconstruction limits the special frequencies used in reconstruction to the Nyquist limit for the data density available and gives better resolution without aliasing. If it is desired, for some *a priori* reason, to limit the spatial frequencies used, this can be done by truncating the transforms of the gradient in Fourier transformation space midway through the calculation. If it is desired to sample a small portion of the available wavefront or decenter it, this may be done with a simple mask operation on the gradient data before the Fourier transformation operation. Unlike Zernike reconstruction methods in which the pupil size and centralization of the pupil is required, such concerns do not effect the fast Fourier transformation.

[0068] Moreover, since the wavefront sensors measure x- and y- components of the gradient map on a regularly spaced grid, the data is band-limited and the data contains no spatial frequencies larger than the Nyquist rate that corresponds to the spacing of the lenslets in the instrument (typically, the lenslets will be spaced no more than about 0.8 mm and about 0.1 mm, and typically about 0.4 mm). Because the data is on a regularly spaced Cartesian grid, non-radial reconstruction methods, such as a Fourier transform, are well suited for the band-limited data.

[0069] In contrast to the Fourier transform, when a series expansion technique is used to generate a wavefront elevation map 100 from the gradient map 92, the gradient map 92 and selected expansion series 96 are used to derive appropriate expansion series coefficients 98. A particularly beneficial form of a mathematical series expansion for modeling the tissue surface are Zernike polynomials. Typical Zernike polynomial sets including terms 0 through 6th order or 0 through 10th order are used. The coefficients a_n for

each Zernike polynomial Z_n may, for example, be determined using a standard least squares fit technique. The number of Zernike polynomial coefficients a_n may be limited (for example, to about 28 coefficients).

[0070] While generally considered convenient for modeling of the optical surface so as to generate an elevation map, Zernike polynomials (and perhaps all series expansions) can introduce errors. Nonetheless, combining the Zernike polynomials with their coefficients and summing the Zernike coefficients 99 allows a wavefront elevation map 100 to be calculated, and in some cases, may very accurately reconstruct a wavefront elevation map 100.

[0071] It has been found that in some instances, especially where the error in the optical tissues of the eye is spherical, the Zernike reconstruction may be more accurate than the Fourier transform reconstruction. Thus, in some embodiments, the modules of the present invention may include both a Fourier transform module 94 and Zernike modules 96, 98, 99. In such embodiments, the reconstructed surfaces obtained by the two modules may be compared by a comparison module (not shown) to determine which of the two modules provides a more accurate wavefront elevation map. The more accurate wavefront elevation map may then be used by 100, 102 to calculate the treatment map and ablation table, respectively.

[0072] In one embodiment, the wavefront elevation map module 100 may calculate the wavefront elevation maps from each of the modules and a gradient field may be calculated from each of the wavefront elevation maps. In one configuration, the comparison module may apply a merit function may be found to determine the difference between each of the gradient maps and an originally measured gradient map. One example of a merit function is the root mean square gradient error, RMS_{grad}, found from the following equation:

$$RMS_{grad} = \sqrt{\frac{\sum_{alldatapo\,\mathrm{int}\,s}\left\{\left(\partial W(x,y)/\partial x - Dx(x,y)^2\right) + \left(\partial W(x,y)/\partial y - Dy(x,y)^2\right)\right\}}{N}}$$

where:

N is the number of locations sampled

(x,y) is the sample location

$\partial W(x,y)/\partial x$ is the x component of the reconstructed wavefront gradient

$\partial W(x,y)/\partial y$ is the y component of the reconstructed wavefront gradient

Dx(x,y) is the x component of the gradient data

Dy(x,y) is the y component of the gradient data

[0073] If the gradient map from the Zernike reconstruction is more accurate, the Zernike reconstruction is used. If the Fourier reconstruction is more accurate, the Fourier reconstruction is used.

[0074] After the wavefront elevation map is calculated, treatment map 102 may thereafter be calculated from the wavefront elevation map 100 so as to remove the regular (spherical and/or cylindrical) and irregular errors of the optical tissues. By combining the treatment map 102 with a laser ablation pulse characteristics 104 of a particular laser system, an ablation table 106 of ablation pulse locations, sizes, shapes, and/or numbers can be developed.

[0075] A laser treatment ablation table 106 may include horizontal and vertical position of the laser beam on the eye for each laser beam pulse in a series of pulses. The diameter of the beam may be varied during the treatment from about 0.65 mm to 6.5 mm. The treatment ablation table 106 typically includes between several hundred pulses to five thousand or more pulses, and the number of laser beam pulses varies with the amount of material removed and laser beam diameters employed by the laser treatment table. Ablation table 106 may optionally be optimized by sorting of the individual pulses so as to avoid localized heating, minimize irregular ablations if the treatment program is interrupted, and the like. The eye can thereafter be ablated according to the treatment table 106 by laser ablation.

[0076] In one embodiment, laser ablation table 106 may adjust laser beam 14 to produce the desired sculpting using a variety of alternative mechanisms. The laser beam 14 may be selectively limited using one or more variable apertures. An exemplary variable aperture system having a variable iris and a variable width slit is described in U.S. Patent No. 5,713,892, the full disclosure of which is incorporated herein by reference. The laser beam may also be tailored by varying the size and offset of the laser spot from an axis of the eye, as described in U.S. Patent No. 5,683,379, and as also described in co-pending U.S. Patent Application Serial Nos. 08/968,380, filed November 12, 1997; and 09/274,999 filed March 22, 1999, the full disclosures of which are incorporated herein by reference.

[0077] Still further alternatives are possible, including scanning of the laser beam over a surface of the eye and controlling the number of pulses and/or dwell time at each location, as described, for example, by U.S. Patent No. 4,665,913 (the full disclosure of which is incorporated herein by reference); using masks in the optical path of laser beam 14 which ablate to vary the profile of the beam incident on the cornea, as described in U.S.

Patent Application Serial No. 08/468,898, filed June 6, 1995 (the full disclosure of which is incorporated herein by reference); hybrid profile-scanning systems in which a variable size beam (typically controlled by a variable width slit and/or variable diameter iris diaphragm) is scanned across the cornea; or the like.

[0078] One exemplary method and system for preparing such an ablation table is described in co-pending U.S. Patent Application S.N. 09/805,737, filed on March 13, 2001 and entitled "Generating Scanning Spot Locations for Laser Eye Surgery," the full disclosure of which is incorporated herein by reference.

[0079] The mathematical development for the surface reconstruction from surface gradient data using a Fourier transform algorithm according to one embodiment of the present invention will now be described. Such mathematical algorithms will typically be incorporated by Fourier transform module 82 (Figure 4), Fourier transform step 94 (Figure 5), or other comparable software or hardware modules to reconstruct the wavefront surface. As can be appreciated, the Fourier transform algorithm described below is merely an example, and the present invention should not be limited to this specific implementation.

[0080] First, let there be a surface that may be represented by the function s(x,y) and let there be data giving the gradients of this surface, $\frac{\partial s(x,y)}{\partial x}$ and $\frac{\partial s(x,y)}{\partial y}$. The goal is to find the surface s(x,y) from the gradient data.

[0081] Let the surface be locally integratable over all space so that it may be represented by a Fourier transform. The Fourier transform of the surface is then given by

$$(1) \qquad \mathcal{F}\,\left(s(x,y)\right)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}s(x,y)e^{-i(ux+vy)}dxdy=S(u,v)$$

[0082] The surface may then be reconstructed from the transform coefficients, S(u,v), using

$$(2) \qquad s(x,y)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}S(u,v)e^{i(ux+vy)}dudv$$

[0083] Equation (2) may now be used to give a representation of the x component of the gradient, $\frac{\partial s(x,y)}{\partial x}$ in terms of the Fourier coefficients for the surface:

$$\frac{\partial s(x,y)}{\partial x}=\frac{\partial\left(\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}S(u,v)e^{i(ux+vy)}dudv\right)}{\partial x}$$

[0084] Differentiation under the integral then gives:

$$(3) \qquad \frac{\partial s(x,y)}{\partial x} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} iuS(u,v) e^{i(ux+vy)} dudv$$

[0085] A similar equation to (3) gives a representation of the y component of the gradient in terms of the Fourier coefficients:

$$(4) \qquad \frac{\partial s(x,y)}{\partial y} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} ivS(u,v) e^{i(ux+vy)} dudv$$

[0086] The x component of the gradient is a function of x and y so it may also be represented by the coefficients resulting from a Fourier transformation. Let the $dx(x,y) = \frac{\partial s(x,y)}{\partial x}$ so that, following the logic that led to (2)

$$(5) \qquad dx(x,y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Dx(u,v) e^{i(ux+vy)} dudv = \frac{\partial s(x,y)}{\partial x}$$

where

$$(6) \qquad F\ (dx(x,y)) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dx(x,y) e^{-i(wx+vy)} dxdy = Dx(u,v)$$

[0087] Equation (3) must equal (5) and by inspecting similar terms it may be seen that in general this can only be true if

$$Dx(u,v) = iuS(u,v)$$

or

$$(7) \qquad S(u,v) = \frac{-iDx(u,v)}{u}$$

[0088] A similar development for the y gradient component using (4) leads to

$$(8) \qquad S(u,v) = \frac{-iDy(u,v)}{v}$$

[0089] Note that (7) and (8) indicate that Dx (u,v) and Dy(u,v) are functionally dependent with the relationship

$$vDx(u,v) = uDy(u,v)$$

[0090] The surface may now be reconstructed from the gradient data by first performing a discrete Fourier decomposition of the two gradient fields, dx and dy to generate

the discrete Fourier gradient coefficients Dx(u,v) and Dy(u,v). From these components (7) and (8) are used to find the Fourier coefficients of the surface S(u,v). These in turn are used with an inverse discrete Fourier transform to reconstruct the surface s(x,y).

[0091] The above treatment makes a non-symmetrical use of the discrete Fourier gradient coefficients in that one or the other is used to find the Fourier coefficients of the surface. The method makes use of the Laplacian, a polynomial, second order differential operator, given by

$$L \equiv \frac{\partial^2}{\partial^2 x} + \frac{\partial^2}{\partial^2 y} \quad \text{or} \quad L \equiv \frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial}{\partial y}\right)$$

[0092] So when the Laplacian acts on the surface function, s(x,y), the result is

$$Ls(x,y) = \frac{\partial^2 s(x,y)}{\partial^2 x} + \frac{\partial^2 s(x,y)}{\partial^2 y} \quad \text{or} \quad Ls(x,y) = \frac{\partial}{\partial x}\left(\frac{\partial s(x,y)}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial s(x,y)}{\partial y}\right)$$

[0093] Using the second form given above and substituting (3) for $\frac{\partial s(x,y)}{\partial x}$ in the first integral of the sum and (4) for $\frac{\partial s(x,y)}{\partial y}$ in the second term, the Laplacian of the surface is found to be

$$Ls(x,y) = \frac{\partial}{\partial x}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} iuS(u,v)e^{i(ux+vy)}dudv\right) + \frac{\partial}{\partial y}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} ivS(u,v)e^{i(ux+vy)}dudv\right)$$

$$Ls(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} -u^2S(u,v)e^{i(ux+vy)}dudv + \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} -v^2S(u,v)e^{i(ux+vy)}dudv$$

$$(9) \qquad Ls(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} -(u^2+v^2)S(u,v)e^{i(ux+vy)}dudv$$

[0094] Equation (9) shows that the Fourier coefficients of the Laplacian of a two dimensional function are equal to $-(u^2 + v^2)$ times the Fourier coefficients of the function itself so that

$$S(u,v) = \frac{-F(Ls(x,y))}{(u^2+v^2)}$$

[0095] Now let the Laplacian be expressed in terms of dx(x,y) and dy(x,y) as defined above so that

$$Ls(x,y) = \frac{\partial}{\partial x}(dx(x,y)) + \frac{\partial}{\partial y}(dy(x,y))$$

and through the use of (5) and the similar expression for dy(x,y)

$$Ls(x,y) = \frac{\partial}{\partial x}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Dx(u,v)e^{i(ux+vy)}dudv\right) + \frac{\partial}{\partial y}\left(\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} Dy(u,v)e^{i(ux+vy)}dudv\right)$$

$$Ls(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} iuDx(u,v)e^{i(ux+vy)}dudv + \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} ivDy(u,v)e^{i(ux+vy)}dudv$$

$$(10) \qquad Ls(x,y) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} i(uDx(u,v) + vDy(u,v))e^{i(ux+vy)}dudv$$

(9) and (10) must be equal and comparing them, it is seen that this requires that:

$$-(u^2 + v^2)S(u,v) = i(uDx(u,v)+vD(u,v))$$

or

$$(11) \qquad S(u,v) = \frac{-i(uDx(u,v) + vDy(u,v))}{u^2 + v^2}$$

[0096] As before, Dx(u,v) and Dy(u,v) are found by taking the Fourier transforms of the measured gradient field components. They are then used in (11) to find the Fourier coefficients of the surface itself, which in turn is reconstructed from them. This method has the effect of using all available information in the reconstruction, whereas the Zernike polynomial method fails to use all of the available information.

[0097] It should be noted, s(x,y) may be expressed as the sum of a new function s(x,y)' and a tilted plane surface passing through the origin. This sum is given by the equation

$$s(x,y) = s(x,y)' + ax + by$$

[0098] Then the first partial derivatives of f(x,y) with respect to x and y are given by

$$\frac{\partial s(x,y)}{\partial x} = \frac{\partial s(x,y)'}{\partial x} + a$$

$$\frac{\partial s(x,y)}{\partial y} = \frac{\partial s(x,y)'}{\partial y} + b$$

[0099] Now following the same procedure that lead to (6), the Fourier transform of these partial derivatives, Dx(u,v) and Dy(u,v), are found to be

$$Dx(u,v)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\partial s(x,y)}{\partial x}e^{-i(wx+vy)}dxdy=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\partial s(x,y)'}{\partial x}e^{-i(wx+vy)}dxdy+\frac{a}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-i(wx+vy)}dxdy$$

$$Dx(u,v)=Dx(u,v)'+a\delta(u,v)) \qquad (12)$$

$$Dy(u,v)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\partial s(x,y)}{\partial y}e^{-i(wx+vy)}dxdy=\frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\partial s(x,y)'}{\partial y}e^{-i(wx+vy)}dxdy+\frac{b}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{-i(wx+vy)}dxdy$$

$$Dy(u,v)=Dy(u,v)'+b\delta(u,v)) \qquad (13)$$

[0100] In (12) and (13), δ(u,v), is the Dirac delta function that takes the value 1 if u = v= 0 and takes the value 0 otherwise. Using (12) and (13) in (11), the expression of the Fourier transform of the surface may be written as

$$S(u,v)=\frac{-i\left(uDx(u,v)'+vDy(u,v)'+(ua+vb)\delta(u,v)\right)}{u^2+v^2}$$

[0101] But it will be realized that the term in the above equation can have no effect whatsoever on the value of S(u,v) because if u and v are not both zero, the delta function is zero so the term vanishes. However in the only other case, u and v are both zero and this also causes the term to vanish. This means that the surface reconstructed will not be unique but will be a member of a family of surfaces, each member having a different tilted plane (or linear) part. Therefore to reconstruct the unique surface represented by a given gradient field, the correct tilt must be restored. The tilt correction can be done in several ways.

[0102] Since the components of the gradient of the tilt plane, a and b, are the same at every point on the surface, if the correct values can be found at one point, they can be applied everywhere at once by adding to the initially reconstructed surface, s(x,y)', the surface (ax +by). This may be done by finding the gradient of s(x,y)' at one point and subtracting the components from the given components. The differences are the constant gradient values a and b. However when using real data there is always noise and so it is best to use an average to find a and b. One useful way to do this is to find the average gradient components of the given gradient field and use these averages as a and b.

$$\langle \partial s/\partial x\rangle = a \qquad\qquad \langle \partial s/\partial y\rangle = b$$

[0103] The reconstructed surface is then given by

$$s(x,y) = s(x,y)' + \langle \partial s/\partial x\rangle\, x + \langle \partial s/\partial y\rangle y \qquad (14)$$

where s(x,y)' is found using the Fourier reconstruction method developed above.

[0104] Attention is now turned to implementation of this method using discrete fast Fourier transform methods. The algorithm employed for the discrete fast Fourier transform in two dimensions is given by the equation

$$(12) \qquad F(k,l) = \sum_{m=1}^{M}\sum_{n=1}^{N} f(n,m) e^{-i2\pi\left(\frac{(k-1)(n-1)}{N}+\frac{(l-1)(m-1)}{M}\right)}$$

with the inverse transform given by

$$(13) \qquad f(n,m) = \frac{1}{MN}\sum_{k=1}^{M}\sum_{l=1}^{N} F(k,l) e^{i2\pi\left(\frac{(k-1)(n-1)}{N}+\frac{(l-1)(m-1)}{M}\right)}$$

[0105] When these equations are implemented N and M are usually chosen so to be equal. For speed of computation, they are usually taken to be powers of 2. (12) and (13) assume that the function is sampled at locations separated by intervals dx and dy. For reasons of algorithmic simplification, as shown below, dx and dy are usually set equal.

[0106] In equation (12) let n be the index of the x data in array f(n,m) and let k be the index of the variable u in the transform array, F(k,l).

[0107] Let us begin by supposing that in the discrete case the x values are spaced by a distance dx, so when equation (12) is used, each time n is incremented, x is changed by an amount dx. So by choosing the coordinate system properly we could represent the position of the pupil data x by:

$$x = (n-1)dx$$

so that:

$$(n-1) = x/dx$$

[0108] Likewise, (m-1) may be set equal to y/dy. Using these relationships, (12) may be written as:

$$F(k,l) = \sum_{m=1}^{M}\sum_{n=1}^{N} f(n,m) e^{-i2\pi\left(\frac{(k-1)x}{Ndx}+\frac{(l-1)y}{Mdy}\right)}$$

[0109] Comparing the exponential terms in this discrete equation with those in its integral form (1), it is seen that

$$u = \frac{2\pi(k-1)}{Ndx} \quad \text{and} \quad v = \frac{2\pi(l-1)}{Mdy}$$

[0110] In these equations notice that Ndx is the x width of the sampled area and Mdy is the y width of the sampled area. Letting

$$\text{(14)} \qquad \begin{array}{l} Ndx = X, \text{ the total x width} \\ Mdy = Y, \text{ the total y width} \end{array}$$

[0111] The above equations become

$$\text{(15)} \qquad u(k) = \frac{2\pi(k-1)}{X} \quad \text{and} \quad v(l) = \frac{2\pi(l-1)}{Y}$$

[0112] Equations (15) allow the Fourier coefficients, Dx(k,l) and Dy(k,l), found from the discrete fast Fourier transform of the gradient components, dx(n,m) and dy(n,m), to be converted into the discrete Fourier coefficients of the surface, S(k,l) as follows.

$$S(k,l) = \frac{-i\frac{2\pi(k-1)}{X}Dx(k,l) - i\frac{2\pi(l-1)}{Y}Dy(k,l)}{\left(\frac{2\pi(k-1)}{X}\right)^2 + \left(\frac{2\pi(l-1)}{Y}\right)^2}$$

[0113] This equation is simplified considerably if the above mentioned N is chosen equal to M and dx is chosen to dy, so that X = Y. It then becomes

$$\text{(16)} \qquad S(k,l) = \left(\frac{-iX}{2\pi}\right)\frac{(k-1)Dx(k,l) + (l-1)Dy(k,l)}{(k-1)^2 + (l-1)^2}$$

[0114] Let us now consider the values u(k) and v(l) take in (13) as k varies from 1 to N and l varies from 1 to M. When k = l =1, u =v = 0 and the exponential takes the value 1. As u and v are incremented by 1 so that k = l = 2, u and v are incremented by unit increments, du and dv, so

$$u(2) = \frac{2\pi}{X} = du \quad \text{and} \quad v(2) = \frac{2\pi}{Y} = dv$$

[0115] Each increment of k or l increments u or v by amounts du and dv respectively so that for any value of k or l, u(k) and v(l) may be written as

$$u(k) = (k-1)du, \quad v(l) = (l-1)dv$$

[0116] This process may be continued until k = N and l = M at which time

$$u(N) = \frac{2\pi(N-1)}{X} = \frac{2\pi(N)}{X} - \frac{2\pi}{X} = \frac{2\pi(N)}{X} - du$$
$$v(M) = \frac{2\pi(M-1)}{X} = \frac{2\pi(M)}{X} - \frac{2\pi}{X} = \frac{2\pi(M)}{X} - dv$$

[0117] But now consider the value the exponential in (13) takes when these conditions hold. In the following, the exponential is expressed as a product

$$e^{\frac{i2\pi(N-1)(n-1)}{N}} e^{\frac{i2\pi(M-1)(m-1)}{M}} = e^{\left(i2\pi(n-1c)-\frac{i2\pi(n-1)}{N}\right)} e^{\left(i2\pi(m-1c)-\frac{i2\pi(m-1)}{M}\right)} = e^{\frac{-i2\pi(n-1)}{N}} e^{\frac{-i2\pi(m-1)}{M}}$$

[0118] Using the same logic as was used to obtain equations (15), the values of u(N) and v(M) are

$$u(N) = -du \qquad \text{and} \qquad v(M) = -dv$$

[0119] Using this fact, the following correlation may now be made between u(k+1) and u(N-k) and between v(l+1) and v(M-l) for k>1 and l>1

$$u(k) = -u(N-k+2) \qquad v(l) = -v(M-l+2)$$

In light of equations (15)

$$(17) \qquad u(N-k+2) = \frac{-2\pi(k-1)}{X} \qquad \text{and} \qquad v(M-l+2) = \frac{-2\pi(l-1)}{Y}$$

[0120] To implement (15), first note that Dx(k,l) and Dy(k,l) are formed as matrix arrays and so it is best to form the coefficients (k-1) and (l-1) as matrix arrays so that matrix multiplication method may be employed to form S(k,l) as a matrix array.

[0121] Assuming the Dx and Dy are square arrays of N x N elements, let the (k-1) array, K(k,l) be formed as a N x N array whose rows are all the same consisting of integers starting at 0 (k = 1) and progressing with integer interval of 1 to the value ceil(N/2)-1. The "ceil" operator rounds the value N/2 to the next higher integer and is used to account for cases where N is an odd number. Then, in light of the relationships given in (17), the value of the next row element is given the value –floor(N/2). The "floor" operator rounds the value N/2 to the next lower integer, used again for cases where N is odd. The row

element following the one with value –floor(N/2) is incremented by 1 and this proceeds until the last element is reached (k=N) and takes the value –1. In this way, when matrix |Dx| is multiplied term by term times matrix |K|, each term of |Dx| with the same value of k is multiplied by the correct integer and hence by the correct u value.

[0122] Likewise, let matrix |L(k,l)| be formed as an N x N array whose columns are all the same consisting of integers starting at 0 (l = 1)and progressing with integer interval of 1 to the value ceil(N/2)-1. Then, in light of the relationships given in (17), the value of the next column element is given the value –floor(N/2). The column element following the one with value –floor(N/2) is incremented by 1 and this proceeds until the last element is reached (l=N) and takes the value –1. In this way, when matrix |Dy| is multiplied term by term times matrix |L|, each term of |Dy| with the same value of l is multiplied by the correct integer and hence by the correct v value.

[0123] The denominator of (15) by creating a matrix |D| from the sum of matrices formed by multiplying, term-by-term, |K| times itself and |L| times itself. The (1,1) element of |D| is always zero and to avoid divide by zero problems, it is set equal to 1 after |D| is initially formed. Since the (1,1) elements of |K| and |L| are also zero at this time, this has the effect of setting the (1,1) element of |S| equal to zero. This is turn means that the average elevation of the reconstructed surface is zero as may be appreciated by considering that the value of (12) when k = l = 1 is the sum of all values of the function f(x,y). If this sum is zero, the average value of the function is zero.

[0124] Let the term-by-term multiplication of two matrices |A| and |B| be symbolized by |A|.*|B| and the term-by-term division of |A| by |B| by |A|./|B|. Then in matrix form, (16) may be written as:

$$(18) \qquad |S| = \left(\frac{-iX}{2\pi}\right)\left(|K|.*|Dx| + |L|.*Dy(k,l)\right)./\left(|K|.*|K| + |L|.*|L|\right)$$

[0125] The common factor $\left(\frac{-iX}{2\pi}\right)$ is neither a function of position nor "frequency" (the variables of the Fourier transform space). It is therefore a global scaling factor.

[0126] As a practical matter when coding (18), it is simpler to form K and L if the transform matrices Dx and Dy are first "shifted" using the standard discrete Fourier transform quadrant shift technique that places u = v= 0 element at location (floor(N/2)+1,floor(N/2)+1). The rows of K and the columns of L may then be formed from

$$\text{row} = [1,2,3,\ldots N-2,N-1,N] - (\text{floor}(N/2)+1)$$
$$\text{column} = \text{row}^T$$

[0127] After the matrix |S| found with (18) using the shifted matrices, |S| is then inverse shifted before the values of s(x,y) are found using the inverse discrete inverse Fourier transform (13).

[0128] The final step is to find the mean values of the gradient fields dx(n,m) and dy(n,m). These mean values are multiplied by the respective x and y values for each surface point evaluated and added to the value of s(x,y) found in the step above to give the fully reconstructed surface.

EXPERIMENTAL RESULTS

[0129] A detailed description of some test methods to compare the surface reconstructions of the expansion series (e.g., Zernike polynomial) reconstruction methods, direct integration reconstruction methods, and Fourier transform reconstruction methods will now be described.

[0130] While not described in detail herein, it should be appreciated that the present invention also encompasses the use of direct integration algorithms and modules for reconstructing the wavefront elevation map. The use of Fourier transform modules, direct integration modules, and Zernike modules are not contradictory or mutually exclusive, and may be combined, if desired. For example, the modules of Figure 5 may also include direct integration modules in addition to or alternative to the modules illustrated. A more complete description of the direct integration modules and methods are described in co-pending U.S. Patent Application S.N. 10/006,992, filed December 6, 2001 and PCT Application No. PCT/US01/46573, filed November 6, 2001, both entitled "Direct Wavefront-Based Corneal Ablation Treatment Program," the complete disclosures of which are incorporated herein by reference.

[0131] To compare the various methods, a surface was ablated onto plastic, and the various reconstruction methods were compared to a direct surface measurement to determine the accuracy of the methods. Three different test surfaces were created for the tests, as follows:

(1) +2 ablation on a 6 mm pupil, wherein the ablation center was offset by approximately 1 mm with respect to the pupil center;

(2) Presbyopia Shape I which has a 2.5 mm diameter "bump," 1.5 μm high, decentered by 1.0 mm.

(3) Presbyopia Shape II which has a 2.0 mm diameter "bump," 1.0 μm high, decentered by 0.5 mm.

[0132] The ablated surfaces were imaged by a wavefront sensor system 30 (*see* Figures 3 and 3A), and the Hartmann-Shack spot diagrams were processed to obtain the wavefront gradients. The ablated surfaces were also scanned by a surface mapping interferometer Micro XCAM, manufactured by Phase Shift Technologies, so as to generate a high precision surface elevation map. The elevation map directly measured by the Micro XCAM was compared to the elevation map reconstructed by each of the different algorithms. The algorithm with the lowest root mean square (RMS) error was considered to be the most effective in reconstructing the surface.

[0133] In both the direct measurement and mathematical reconstruction, there may be a systematic "tilt" that needs correction. For the direct measurement, the tilt in the surface (that was introduced by a tilt in a sample stage holding the sample) was removed from the data by subtracting a plane that would fit to the surface.

[0134] For the mathematical reconstructions, the angular and spatial positions of the surface relative to the lenslet array in the wavefront measurement system introduced a tilt and offset of center in the reconstruction surfaces. Correcting the "off-center" alignment was done by identifying dominant features, such as a top of a crest, and translating the entire surface data to match the position of this feature in the reconstruction.

[0135] To remove the tilt, in one embodiment a line profile of the reconstructed surface along an x-axis and y-axis were compared with corresponding profiles of the measured surface. The slopes of the reconstructed surface relative to the measured surface were estimated. Also the difference of the height of the same dominant feature (e.g., crest) that was used for alignment of the center was determined. A plane defined by those slopes and height differences was subtracted from the reconstructed surface. In another embodiment, it has been determined that the tilt in the Fourier transform algorithm may come from a DC component of the Fourier transform of the x and y gradients that get set to zero in the reconstruction process. Consequently, the net gradient of the entire wavefront is lost. Adding in a mean gradient field "untips" the reconstructed surface. As may be appreciated, such methods may be incorporated into modules of the present invention to remove the tilt from the reconstructions.

[0136] A comparison of reconstructed surfaces and a directly measured surface for a decentered +2 lens is illustrated in Figure 6. As illustrated in Figure 6, all of the

reconstruction methods matched the surface well. The RMS error for the reconstructions are as follows:

Fourier	0.2113e-3
Direct Integration	0.2912e-3
Zernike (6th order)	0.2264e-3

[0137] Figure 7 shows a cross section of the Presbyopia Shape I reconstruction. As can be seen, the Zernike 6th order reconstruction excessively widens the bump feature. The other reconstructions provide a better match to the surface. The RMS error for the four reconstruction methods are as follows:

Fourier	0.1921e-3
Direct Integration	0.1849e-3
Zernike (6th order)	0.3566e-3
Zernike (10th order)	0.3046e-3

[0138] Figure 8 shows a cross section of Presbyopia Shape II reconstruction. The data is qualitatively similar to that of Figure 7. The RMS error for the four reconstruction methods are as follows:

Fourier	0.1079e-3
Direct Integration	0.1428e-3
Zernike (6th order)	0.1836e-3
Zernike (10th order)	0.1413e-3

[0139] From the above results, it appears that the 6th order Zernike reconstructions is sufficient for smooth surfaces with features that are larger than approximately 1-2 millimeters. For sharper features, however, such as the bump in the presbyopia shapes, the 6th order Zernike reconstruction gives a poorer match with the actual surface when compared to the other reconstruction methods.

[0140] Sharper features or locally rapid changes in the curvature of the corneal surface may exist in some pathological eyes and surgically treated eyes. Additionally, treatments with small and sharp features may be applied to presbyopic and some highly aberrated eyes.

[0141] Applicants believe that part of the reason the Fourier transformation provides better results is that, unlike the Zernike reconstruction algorithms (which are defined

over a circle and approximates the pupil to be a circle), the Fourier transformation algorithm (as well as the direct integration algorithms) makes full use of the available data and allows for computations based on the actual shape of the pupil (which is typically a slight ellipse). The bandwidth of the discrete Fourier analysis is half of the sampling frequency of the wavefront measuring instrument. Therefore, the Fourier method may use all gradient field data points. Moreover, since Fourier transform algorithms inherently have a frequency cutoff, the Fourier algorithms filter out (i.e., set to zero) all frequencies higher than those that can be represented by the data sample spacing and so as to prevent artifacts from being introduced into the reconstruction such as aliasing. Finally, because many wavefront measurement systems sample the wavefront surface on a square grid and the Fourier method is performed on a square grid, the Fourier method is well suited for the input data from the wavefront instrument.

[0142] In contrast, the Zernike methods use radial and angular terms (e.g., polar), thus the Zernike methods weigh the central points and the peripheral points unequally. When higher order polynomials are used to reproduce small details in the wavefront, the oscillations in amplitude as a function of radius are not uniform. In addition, for any given polynomial, the meridional term for meridional index value other than zero is a sinusoidal function. The peaks and valleys introduced by this Zernike term is greater the farther one moves away from the center of the pupil. Moreover, it also introduces non-uniform spatial frequency sampling of the wavefront. Thus, the same polynomial term may accommodate much smaller variations in the wavefront at the center of the pupil than it can at the periphery. In order to get a good sample of the local variations at the pupil edge, a greater number of Zernike terms must be used. Unfortunately, the greater number of Zernike terms may cause over-sampling at the pupil center and introduction of artifacts, such as aliasing. Because Fourier methods provide uniform spatial sampling, the introduction of such artifacts may be avoided.

[0143] Additional test results on clinical data are illustrated in Figures 9 to 11. A Fourier method of reconstructing the wavefront was compared with 6th order Zernike methods and a direct integration method to reconstruct the wavefront from the clinical data. The reconstructed wavefronts were then differentiated to calculate the gradient field. The root mean square (RMS) difference between the calculated and the measured gradient field was used as a measure of the quality of reconstruction.

[0144] The test methods of the reconstruction were as follow: A wavefront corresponding to an eye with a large amount of aberration was reconstructed using the three

algorithms (e.g., Zernike, Fourier, and direct integration). The pupil size used in the calculations was a 3 mm radius. The gradient field of the reconstructed wavefronts were compared against the measured gradient field. The x and y components of the gradient at each sampling point were squared and summed together. The square root of the summation provides information about the curvature of the surface. Such a number is equivalent to the average magnitude of the gradient multiplied by the total number of sampling points. For example, a quantity of 0 corresponds to a flat or planar wavefront. The ratio of the RMS deviation of the gradient field with the quantity gives a measure of the quality of reconstruction. For example, the smaller the ratio, the closer the reconstructed wavefront is to the directly measured wavefront. The ratio of the RMS deviations (described *supra*) with the quantity of the different reconstructions are as follows:

Zernike (6th order)	1.09
Zernike (10th order)	0.82
Direct integration	0.74
Fourier	0.67

[0145] Figure 9 illustrates a vector plot of the difference between the calculated and measured gradient field. The Zernike plot (noted by "Z field") is for a reconstruction using terms up to the 10th order. Figure 11 illustrates that the Zernike reconstruction algorithm using terms up to 6th order is unable to correctly reproduce small and sharp features on the wavefront. As shown in Figure 10, Zernike algorithm up to the 10th order term is better able to reproduce the small and sharp features. As seen by the results, the RMS deviation with the measured gradient is minimum for the Fourier method.

[0146] A variety of modifications are possible within the scope of the present invention. For example, the Fourier based methods of the present invention may be used in the aforementioned ablation monitoring system feedback system for real-time intrasurgical measurement of a patient's eye during and/or between each laser pulse. The Fourier-based methods of the present invention are particularly well suited for such use due to its measurement accuracy and high speed. Hence, the scope of the present application is not limited to the specifics of the embodiments described herein, but is instead limited solely by the appended claims.